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THE AREA MOMENT OF INERTIA OF THE TIBIA: A RISK FACTOR FOR STRESS FRACTURES

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Abstract—In a prospective study of stress fractures among Israeli infantry recruits, the area moment of inertia of the tibia was found to have a statistically significant correlation with the incidence of tibial, femoral and total stress fractures. Recruits with "low" area moments of inertia of the tibia were found to have higher stress fracture morbidity than those with "high" area moments of inertia. The best correlation was obtained when the area moment of inertia was calculated about the AP axis of bending at a cross-sectional level corresponding to the narrowest tibial width on lateral X-rays, a point which is at the distal quarter of the tibia. This finding indicates that bending forces about the approximate AP axis are an important causal factor for tibial and many other stress fractures. The bone's bending strength, or ability to resist bending moments, as measured by the area moment of inertia, helps determine risk to stress fracture.

NOMENCLATURE

A	tibial width in the medial–lateral direction
B	tibial width in the anterior–posterior direction
C_m	width of medial tibial cortex
C_l	width of lateral tibial cortex
C_a	width of anterior tibial cortex
C_p	width of posterior tibial cortex
H	area of the inner ellipse (or "hole") of the tibial cross-section
S	area of the outer ellipse of the tibial cross-section
X_0, Y_0	coordinates of the tibial cross-section's center of gravity
I_{ML1}	Area moment of inertia of the tibia about a medial–lateral axis at cross-sectional level 1
I_{ML2}	area moment of inertia of the tibia about a medial–lateral axis at cross-sectional level 2
I_{AP1}	area moment of inertia of the tibia about an anterior axis at cross-sectional level 1
I_{AP2}	area moment of inertia of the tibia about an anterior axis at cross-sectional level 2

INTRODUCTION

Stress fractures are a major problem among runners and military recruits (Belkin, 1980; Gilbert and Johnson, 1966; Milgrom *et al*, 1985). The extrinsic demands of a training program place the training population at risk to develop stress fractures. What determines whether or not an individual trainee develops a stress fracture is the interaction of these extrinsic training factors with factors intrinsic to his own body (Giladi *et al*., 1985).

During training the lower extremity is subjected to repetitive tension, compression, torsion and bending forces (Lanyon *et al*., 1975; Lovejoy *et al*., 1976), as well as muscle traction forces. Depending on the magnitude and repetition of these forces, they can either act singly or in combination to cause stress fractures. The bone's ability to resist such forces is dependent, among other factors, on its geometry (Lovejoy *et al*., 1976).

We have previously reported the association between the geometric parameter, tibial bone width, and the incidence of stress fractures (Giladi *et al*., 1987). Soldiers with narrow tibias had a higher incidence of stress fractures than those with wide tibias. We theorized that the significance of tibial width as a risk factor for stress fractures was based on its biomechanical relationship to the area moment of inertia. The bending strength of bone is a function of the area moment of inertia about the axis of bending for the particular cross-section of bone studied (Frankel and Burstein, 1971).

As part of a prospective study of military stress fractures, we undertook to determine the relationship between the area moments of inertia of the tibia at several levels and the incidence of tibial stress fractures, as well as stress fractures in other sites. We report here these results and integrate them with our previous observations on tibial bone width as a risk factor for stress fractures.

MATERIALS AND METHODS

Patient population

A group of 295 male recruits from selected infantry units in the Israeli Army was evaluated in a prospective study of possible risk factors for stress fractures. All of the participants in the study signed an informed consent which included a description of the goals of the study.

Each of the recruits had a detailed pre-training screening. The methods of several subsections of this study have been reported previously (Giladi *et al*.,

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1987; Milgrom *et al.*, 1985). The screening included the following.

(1) Measurement of weight and height.

(2) AP X-rays of the tibias with the feet positioned in 15 degrees of internal rotation and standard lateral X-rays of the tibias. The tube-film distance was 90 cm. Measurements of total tibial and cortical widths in the anterior–posterior plane and in the medial–lateral plane were made on these X-rays at two different levels (Fig. 1): at the point of the narrowest tibial width on the AP X-rays (level 1) and at the point of the narrowest tibial width on lateral X-rays (level 2). Measurements were made using a 7 × Peak magnification ruler which measures to the nearest 0.1 mm and were done by one person.

During the course of 14 weeks of basic training the recruits had mandatory check-ups every 3 weeks by a team of army doctors in the field. They were questioned about symptoms compatible with stress fractures and those with complaints had physical examinations. The recruits had free access to the medical staff on a daily basis and were encouraged to report symptoms. Recruits with symptoms compatible with stress fractures were given 3 days rest and if still symptomatic on return to activity were sent to the hospital-based orthopaedist.

The recruit's places of pain were recorded and measured from anatomical landmarks. Appropriate X-rays were taken and late phase Tc 99 MDP scintigraphy routinely performed. A diagnosis of a stress fracture was made on the basis of positive X-rays and/or a positive scintigram, using the criteria of Prather *et al.* (1977). Scintigraphy was considered to be diagnostic of a stress fracture when a focal area of increased uptake was found in the absence of other bony pathology.

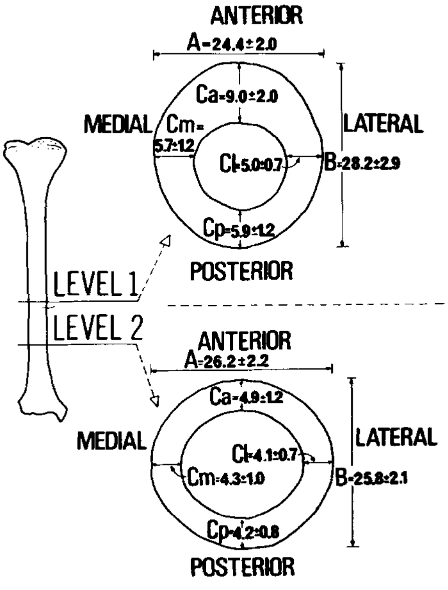

Fig. 1. The two levels at which tibial cross-sections were derived on the basis of measurements taken from X-rays. The mean values of each of the cortical and tibial width measurements for the 286 recruits is shown.

Calculation of the area moment of inertia of the tibia

For purposes of calculation the tibial cross-section was idealized as an elliptical ring with an eccentric hole (Fig. 1). The area moment of inertia (I) of the cross-section was calculated about two axes of bending: the medial–lateral (I_{ML}) and the anterior–posterior (I_{AP}). These calculations were done for the two cross-sections corresponding to the points of narrowest tibial widths on the AP (level 1) and lateral X-rays respectively (level 2). The following direct measurements were made on the X-rays at the two cross-sectional levels for both the left and right tibias in 286 out of the 295 recruits (Fig. 1):

A	tibial width in the medial–lateral direction
B	tibial width in the anterior–posterior direction
C_m	width of the medial tibial cortex
C_l	width of the lateral tibial cortex
C_a	width of the anterior tibial cortex
C_p	width of the posterior tibial cortex.

From these the area moments of inertia I_{ML1} and I_{AP1} at level 1 and I_{ML2} and I_{AP2} at level 2 were calculated. Both axes of rotation are through the cross-section's centroid. The derivation of the expression for the area moments of inertia is given in Appendix 1 and Fig. 2. The mean area moment of inertia of the right and left tibias for each level and axis was used in calculating significance with stress fracture incidence.

In four subjects CT scan sections were obtained of the tibia at levels corresponding to levels 1 and 2 on the plain X-rays. The error in the CT sections was found to be less than one per cent when they were compared with phantom images. The tibial cross-sections were drawn at levels 1 and 2 on the basis of the cortical and tibial width measurements taken from plain X-rays. Figure 3 shows the drawings of these derived cross-sections superimposed on the corres-

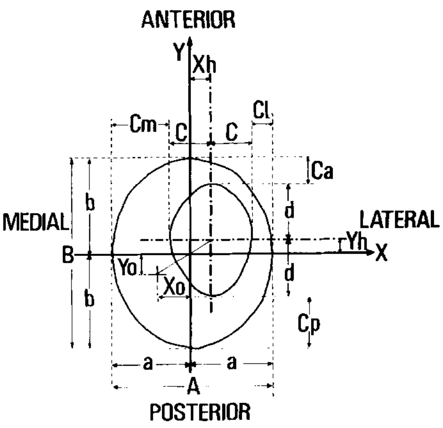

Fig. 2. Graphic representation of the method used to determine the center of gravity and the area moments of inertia for each cross-section based on X-rays.

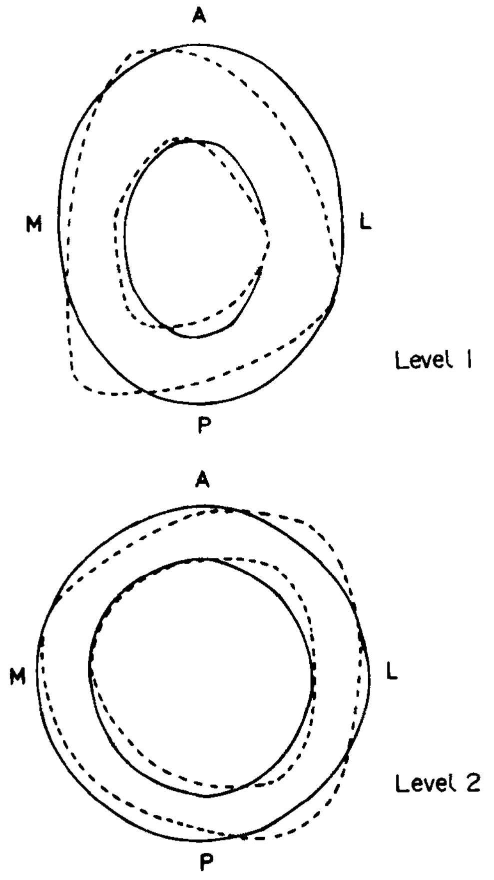

Fig. 3. The tibial cross-section as derived from calculations taken from X-rays is superimposed on the CT section of the corresponding level.

ponding CT section. In all four subjects it was found that the model of the tibial cross-section as an elliptical ring with an eccentric hole was more accurate at level 2 than for level 1. This finding is similiar to that previously noted by Ruff and Hayes (1983).

Statistical analysis

A standard evaluation form for data collection was designed before the study was begun. Statistics were calculated with the Statistical Package for the Social Sciences (SPSS) and the Statistical Analysis System (SAS).

The frequency distribution histograms of tibial bone width and area moment of inertia were found to be skewed and not normally distributed. Therefore non-parametric statistical tests were used in the analysis. Associations between bone width, area moment of inertia and stress fractures were examined using the ranks of variables, instead of the original values, using the stepwise logistic regression tests. The Wilcoxon rank-sum test was used to examine differences in each of the variables, between recruits with and without stress fractures. The chi-square test was used to examine differences in stress fracture morbidity rates among recruits grouped according to high and low area moments of inertia of the tibia.

RESULTS

During the course of basic training 91 out of 295 recruits (31%), were found to have stress fractures by scintigraphic criteria (Milgrom *et al.*, 1985). A total of 184 individual stress fracture sites were found. Fifty one per cent of the fractures were in the tibial diaphysis, 5% in the tibial plateau, 21% in the femoral diaphysis, 9% in the femoral supracondylar region and 4% in the femoral condyles. Only 9% of the fractures were in the feet. All of the tibial and femoral diaphyseal stress fractures occurred in the medial cortex.

Tibial bone width in the medial–lateral direction, as measured on X-rays at cross-sectional levels 1 and 2 (Fig. 1), had a statistically significant correlation with the incidence of femoral, tibial and total stress fractures (Giladi *et al.*, 1987). Recruits with wider tibias had less stress fractures than those with narrow tibias. Level 1 was located 13.7 cm (mean) above the ankle joint; level 2 was 8.7 cm (mean) above the ankle joint. Figure 1 shows the mean tibial and cortical widths in the anterior–posterior and medial–lateral plane for the two cross-sectional levels.

Table 1 compares the mean area moments of inertia of recruits with and without stress fractures for each of the four area moments of inertia. For I_{AP1}, I_{AP2} and I_{ML2} the difference between the groups with and without femoral, tibial and total stress fractures was found to be significant. Using stepwise logistic regression the I_{AP2} was found to be the best predictor of risk for femoral ($p<0.001$), tibial ($p=0.016$) as well as for total stress fractures ($p=0.004$). According to the stepwise logistic regression analysis none of the other measurements of area moment of inertia added to the predictability of risk for stress fractures given by I_{AP2} alone.

The recruit population was divided into three (each with an equal number of recruits), corresponding to those with the lowest area moments of inertia, those with intermediate values and those with high area moments of inertia. Stress fracture incidence was calculated for each group and it was found that the group with the lowest area moments of inertia had the highest morbidity. Table 2 shows the incidence rates of stress fractures among recruits with low area moments of inertia (the lowest third of the population) and recruits with high area moments of inertia (the upper two thirds of the population). The difference between the two groups is significant for femoral ($p<0.001$), tibial ($p<0.001$) and total stress fractures ($p<0.001$) by the chi-square test.

By logistic regression it was found that the area moment of inertia I_{AP2} was a better predictor of tibial stress fracture risk than a measurement of tibial width in the medial–lateral direction at either level 1 or 2

Table 1. Mean area moment of inertia of the tibia of recruits with and without stress fractures

	Cross-section and axis of area moment of inertia (mm^4)			
Stress fractures	Mean I_{ML1}	Mean I_{ML2}	Mean I_{AP1}	Mean I_{AP2}
Femur				
with S.F. ($n=36$)	23,933±8643	15,984±5413	15,359±4107	15,240±4177
without S.F. ($n=250$)	25,669±8260	18,238±4859	18,923±5201	18,555±4586
p value	0.242	< 0.001	< 0.001	< 0.001
Tibia				
with S.F. ($n=58$)	24,684±8738	16,780±4592	17,467±5697	16,830±4638
without S.F. ($n=228$)	25,646±8211	18,253±5038	18,730±5054	18,470±4619
p value	0.433	0.036	0.036	0.007
All types				
with S.F. ($n=86$)	25,114±8928	17,022±4939	17,397±5488	16,925±4787
without S.F. ($n=200$)	25,596±8053	18,355±4954	18,939±5022	18,659±4520
p value	0.654	0.016	0.005	< 0.001

p values according to Wilcoxon rank-sum test.

Table 2. Stress fracture morbidity rates among recruits with low and high area moments of inertia of the tibia (I_{AP2})

	Area moment of inertia (I_{AP2})				
	Low ($n=95$) 7330–15,550 mm^4		High ($n=191$) 15,551–32,500 mm^4		
Fracture location	No. Recruits	(%)	No. Recruits	(%)	p value
Femur	23	(24.2)	13	(6.8)	<0.001
Tibia	30	(31.6)	28	(14.6)	<0.001
Total stress fractures	41	(43.2)	45	(23.6)	<0.001

p values according to chi-square test.

($p=0.008$). For femoral stress fractures this significance was reversed, with tibial bone width in the medial–lateral direction at either level 1 or 2 being the best predictor of risk ($p<0.001$). For total stress fractures the best predictor of risk was I_{AP2} ($p=0.002$).

A recruit's weight or height had no statistically significant relationship with the incidence of stress fractures.

DISCUSSION

The concept that risk factors for stress fractures might exist in a population was hypothesized by Giladi *et al.* (1985). They proposed that stress fractures might be similar to other disease entities such as cardiovascular disease in that there are both intrinsic and extrinsic risk factors for disease.

This hypothesis was studied in a prospective study of possible risk factors for stress fractures in the Israeli Army, parts of which have been reported (Giladi *et al.*, 1987; Margulies *et al.*, 1986; Milgrom *et al.*, 1985). Initially tibial bone width in the medial–lateral plane was identified as having a highly significant statistical correlation with the incidence of stress fractures (Giladi *et al.*, 1987). The biomechanical and/or physiological basis for this factor's significance was not studied in the first phases of the study. It was, however, hypothesized that the biomechanical explanation might lie in the relationship between tibial bone width and the area moment of inertia of the tibia.

The area moment of inertia is a measure of the resistance to bending load of a body of material (Frankel and Burstein, 1971). For a cylindrical geometric configuration, calculating the area moment of inertia is relatively simple. All cross-sections are uniform along the length; the radius of the cross-section is the same about any axis of bending; the geometric center of the cross-section coincides with the inertial center of the material. The area moment of inertia for a cylinder is proportional to the radius. For a complex configuration, such as the tibia, calculation is more difficult (Miller and Purkey, 1980; Minns *et al.*, 1975).

The tibia has varying cross-sectional shapes along its length (Lovejoy *et al.*, 1976; Ruff and Hayes, 1983). In this study we calculated the area moments of inertia at two levels. Level 1 corresponds to the approximate middle third of the tibia; level 2 is approximately at the distal quarter of the tibia. To calculate the area moment of inertia on the basis of AP and Lateral X-rays of the tibia we idealized the tibia as an elliptical ring with an eccentric hole (Fig. 2). A comparison of measurements of the area moments of inertia calculated from X-rays with those measured directly on CT sections (Fig. 3) show the correctness of this model for level 2. For level 1 the outer tibial shape approaches a triangle.

The area moments of inertia in this study were calculated about anterior–posterior and medial–lateral axes of bending. Bending about the anterior–posterior axis places maximum stresses on the medial or lateral cortices; bending about the medial–lateral axis places maximum stresses on the anterior or posterior cortices. These axes were chosen because the area moments of inertia were derived from measurements taken from AP and lateral X-rays. There was no possibility in this study of measuring moments about any oblique axes, even though these may have more accurately represented the *in vivo* bending axis.

Stress fractures are a local phenomena. They occur where the stresses are largest in relationship to the bone's strength. The diaphyseal tibial and femoral stress fractures in this study all occurred in the medial cortex. The stress on the medial cortex in response to a bending load is determined by the formula (Lovejoy *et al.*, 1976):

$$\sigma = \frac{MY}{I_{ap}},$$

where

σ = stress
M = the bending moment
Y = the distance from the centroid axis to the most distant fiber from that axis
I_{ap} = the area moment of inertia about the AP axis of bending.

According to this formula the larger the area moment of inertia the smaller are the stresses on the medial cortex.

Three tibial area moment of inertia measurements (I_{AP1}, I_{AP2} and I_{ML2}) were found to have a statistically significant relationship with the incidence of stress fractures. Recruits with high area moments of inertia had less stress fractures than those with low area moments of inertia. This suggests that bending forces play a significant role in the development of lower extremity stress fractures. The best predictor of risk for tibial, femoral and total stress fractures of all the measurements was that made at level 2 about the AP axis of bending (I_{AP2}). That this measurement is the most statistically significant probably reflects two facts: (1) At this cross-sectional level the actual geometry approaches the model of an elliptical ring with an eccentric hole used for our calculations; (2) most stress fractures in the tibia and femur occurred in the medial cortex and if bending forces are important in producing them they would occur about an approximate AP axis of bending.

By logistic regression analysis it was found that the area moment of inertia of the tibia as measured at I_{AP2} was a better predictor of risk for tibial stress fractures than a measurement of tibial bone width in the medial–lateral direction at either levels 1 or 2. This additional finding reinforces the concept of bending forces as a major causal factor in tibial stress fractures.

For femoral stress fractures the tibial bone width in the medial–lateral direction was a better predictor of risk than the tibial area moment of inertia, even though the latter was highly significant ($p < 0.001$). To determine the true relationship between femoral stress fractures and the area moment of inertia, measurements of the femoral area moment of inertia will have to be made in a future study.

In an attempt to ascertain whether bone mineralization, in addition to bone geometry, influences the incidence of stress fractures the bone mineral content of each recruit's tibias was measured prior to training (Margulies *et al.*, 1986). As reported previously, no statistically significant relationship between bone mineral content and stress fracture incidence was found.

The finding that the area moment of inertia of the tibia is a predictor of risk for stress fractures has several implications: (1) it helps explain mechanistically our previous observation that tibial bone width measurements can predict risk to stress fractures; (2) it shows that bending forces are an important causal factor for tibial stress fractures and perhaps for femoral stress fractures; (3) it lends credence to the hypothesis that stress fracture is a disease entity with risk factors; (4) it gives us a picture that the individual with wide tibias, with a high area moment of inertia, is relatively protected from stress fractures; his bones are biomechanically superior in their bending strength and probably other strength modalities as well. This strength property coupled with the bone's reparative ability would seem to determine whether or not he develops stress fractures during training.

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APPENDIX 1

$a = A/2$

$b = B/2$

$$c = a - \frac{C_m + C_l}{2}$$

$$d = b - \frac{C_a + C_p}{2}$$

S = area of the outer ellipse of the tibial cross-section

$S = \P ab$

H = area of the inner ellipse or ("hole") of the tibial cross-section

$H = \P cd$

$X_h = a - (C_l + c)$

$Y_h = b - (C_a + d)$

X_0, Y_0 = coordinates of the tibial cross-section's center of gravity

$$X_0 = -\frac{HX_h}{S-H} = -\frac{H}{S-H}[a-(C_l+c)\}$$

$$Y_0 = -\frac{H}{S-H}[b-(C_a+d)]$$

$$I_{ML} = I_{rx(X_0,\,Y_0)} = \P ab\left(\frac{b^2}{4} + Y_0^2\right) - \P cd\left[\frac{d^2}{4} + (Y_h - Y_0)^2\right]$$

$$I_{AP} = I_{ry(X_0,\,Y_0)} = \P ab\left(\frac{a^2}{4} + X_0^2\right) - \P cd\left[\frac{c^2}{4} + (X_h - X_0)^2\right].$$